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Unsteady Couette Flow of a Bingham Fluid in Contact with a Jeffrey Fluid

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Abstract

In this paper, unsteady Couette flow of a Bingham fluid in contact with a Newtonian fluid in a channel bounded by parallel plates is investigated. The flow region is divided in to lower and upper regions. The lower and upper regions are filled with Bingham and Jeffrey fluids respectively. The velocity fields in the lower and upper regions are determined and the results are discussed through graphs.

Keywords: Unsteady Couette flow, Bingham fluid, Jeffrey fluid.

Introduction

The study of two fluid flows in a channel is important in connection with plastics manufacture processing of food stuffs and movement of biological fluids in physiological systems. Biofluids such as blood cannot be considered as single component systems. In view of this, blood is modeled by many researchers either as a combination of two immiscible Newtonian fluids or as a combination of two non-Newtonian fluids or Newtonian and non Newtonian fluids. Further Bugliarello and Sevillo [1] and some others confirmed through experiments that blood has to be treated as an yield stress fluid. Among the several yield stress fluid models available, Bingham model is a simple model containing the effect of yield stress and this reduces to classical Newtonian fluid model in the absence of yield stress. In view of the complex behavior of blood in the circulatory system, it is necessary to consider the biofluid blood as a two layered fluid. Therefore the study of Bingham fluid in contact with a Newtonian fluid becomes important and it has potential applications in the design of pumps used in engineering and medicine.

Bird et al. [2] investigated the Bingham fluid in a rigid circular tube. Rathy [3] studied the flow of a Bingham fluid in a channel and in an annulus with impermeable walls. Sai [4] investigated the unsteady flow of a viscous incompressible fluid over a naturally permeable bed. The unsteady flow of two immiscible conducting fluids between two parallel plates is studied by Mitra [5]. Vajravelu et al. [6] made a study on the Bingham fluid flow in a circular tube with permeable wall. The Bingham fluid flow between two permeable beds is discussed by Govardhan et al. [7]. Ravana et al [8] studied the free surface flow of a Bingham fluid in an in claimed channel over a permeable bed. The problem of rotational motion of a Bingham fluid in the gap between two coaxial cylinders, the outer one being at rest and the inner one moving at a given angular velocity, is solved by Comparini [9]. Narahari et al. [10] studied the unsteady flow of a Bingham fluid between two permeable beds. Sankara Reddy et al. [11] made a detailed study on the Bingham fluid flow in an inclined channel bounded by two permeable beds. Hayat et al. [12] discussed the unsteady Couette flow of a second grade fluid in a porous layer. Sokrates Tsangaris et al. [13] studied Couette flow of a Bingham plastic in a channel with equally porous parallel walls.

The unsteady flow of a viscous incompressible fluid is of considerable interest in a biomechanics and medicine. Sir Isaac Newton showed that stress and the rate of strain are linearly related for many familiar fluids such as water and air. These Newtonian fluids are modeled through a coefficient called viscosity, which depends on the specific fluid. However, some of the other materials, such as emulsions and slurries and some visco-elastic materials (e.g. blood, some polymers), have more complicated non-Newtonian stress-strain relationships. These materials include sticky liquids such as latex, honey and lubricants which are studied in the sub-discipline of rheology. Non-Newtonian models like Bingham model, Hershel-Bulkley model, Casson model. Jeffery models etc. are proposed by researchers to describe the flow behavior of the above materials. In view of this it will be interesting to study the flow of an yield stress fluid in contact with a Newtonian fluid.

In this paper, unsteady Couette flow of a Bingham fluid in contact with a Newtonian fluid in a channel bounded by parallel plates is investigated. The flow region is divided in to lower and upper regions. The lower and upper regions are filled with Bingham and Jeffrey fluids respectively. The velocity fields in the two regions are determined and the results are discussed through graphs.

Mathematical Formulation and Solution

Consider the unsteady Couette flow of two immiscible fluids between two parallel plates (see Figure1). The lower plate is at rest and the upper plate is moving constant velocity U. X-axis is taken along the lower stationary plate and Y-axis is taken perpendicular to X-axis. The flow region between the plates is divided into two regions. The region between y=0 and y=h_1 consists of Bingham fluid and the region between y=h_1 and y=h consists of Jeffrey fluid. Since Bingham fluid is an yield stress fluid, the region $0 \le y \le h_1$ is further divided into regions $0 \le y \le y_1,$ $y_1 \le y \le y_2$ and $y_2 \le y \le h$. The region $y_1 \le y \le y_2$ represents the plug flow region. The following assumptions are made in the analysis of the problem:

a) The flow in the x-direction is driven by an exponentially time dependent pressure gradient.
b) The flow is unsteady and fully developed so that all physical characteristics except pressure are functions of 'y' and 't' only.
c) The velocity field and the pressure distribution vary exponentially with time.

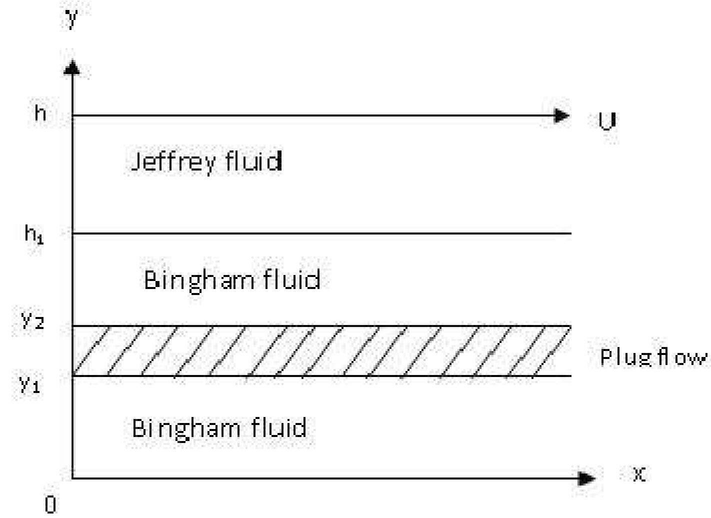

Fig 1: Physical Model

In view of the above assumptions, the basic equations reduce to

Lower Region I $\left(0 \le y \le h_1\right)$

$$\rho\frac{\partial u_i}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial T}{\partial y} \tag{1}$$

where

$$T = \mu_1\frac{\partial u_i}{\partial y} + \sigma_0 \text{sign}\left(\frac{\partial u_i}{\partial y}\right) \quad |T| > \sigma_0$$

$$\frac{\partial u_i}{\partial y} = 0, \qquad |T| < \sigma_0, \qquad (i = 1, 2) \tag{2}$$

Upper Region II

$$\rho\frac{\partial u_3}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\frac{\mu_2}{1+\lambda_1}\frac{\partial u_3}{\partial y}\right) \tag{3}$$

The boundary conditions are given by

$T = T_1$ at y = 0 (4a)

$u_1 = 0$ at y = 0 (4b)

$u_1\,(y_1) = u_2\,(y_2)$ (4c)

$$\left.\frac{du_1}{dy}\right]_{y=y_1} = \left.\frac{du_2}{dy}\right]_{y=y_2} \tag{4d}$$

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$u_2 = u_3$ at $y = h_1$ (4e)

$$\mu_1 \frac{du_2}{dy} - \sigma_0 = \frac{\mu_2}{1+\lambda_1}\frac{du_3}{dy} \quad \text{at } y = h_1 \tag{4f}$$

$u_3 = u_0$ at $y = h$ (4g)

where u_1, u_2, u_3 are velocity components in the Zones I, II, III respectively, p is the pressure, T is the shear stress and T_1 is the shear stress at y=0.
In view of assumption (c), it follows that

$$\frac{\partial p}{\partial x} = -\rho e^{\lambda^2 t}$$

$$u_i(y,t) = s_i(y)e^{\lambda^2 t}, \text{ i=1, 2, 3}$$

$$T = \tau e^{\lambda^2 t}, \qquad T_1 = \tau_1 e^{\lambda^2 t}$$

$$\sigma_0 = \tau_0 e^{\lambda^2 t}, \quad U_0 = Ue^{\lambda^2 t} \tag{5}$$

Using (5), the governing equations (1) to (3) become,

Lower Region I

$$\rho\lambda^2 S_i = \rho + \frac{d\tau}{dy} \tag{6a}$$

where

$$\tau = \mu_1 \frac{ds_i}{dy} + \tau_0 \text{sign}\left(\frac{ds_i}{dy}\right), \quad |\tau| > \tau_0 \tag{6b}$$

$$\frac{ds}{dy} = 0, \qquad |\tau_1| < \tau_0 \tag{7}$$

Upper Region II

$$\rho\lambda^2 s_3 = \rho + \frac{\mu_2}{1+\lambda_1}\frac{d^2 s_3}{dy^2} \tag{8}$$

The boundary conditions are given by

$\tau = \tau_1$ at $y = 0$ (9a)

$s_1 = 0$ at $y = 0$ (9b)

$s_1(y_1) = s_2(y_2)$ (9c)

$$\left.\frac{ds_1}{dy}\right]_{y=y_1} = \left.\frac{ds_2}{dy}\right]_{y=y_2} = 0 \tag{9d}$$

$s_2 = s_3$ at $y = h_1$ (9e)

$$\mu_1 \frac{ds_2}{dy} - \tau_0 = \frac{\mu_2}{1+\lambda_1}\frac{ds_3}{dy} \quad \text{at } y = h_1 \tag{9f}$$

$s_3 = U$ at $y = h$ (9g)

It is convenient to introduce the following non-dimensional quantities:

$$\bar{y} = \frac{y}{h}, \bar{h}_1 = \frac{h_1}{h}, \qquad \bar{s}_i = \frac{s_i}{u_p}, \qquad \bar{\tau} = \frac{\tau h}{\mu_1 u_p},$$

$$u_p = \frac{h^2 p}{\mu_1}, \tag{10}$$

In view of the above non-dimensional quantities, the basic equations (6) to (8) and the boundary conditions (9a) to (9g) can be expressed in non-dimensional form, dropping bars, as:

Region I: ($0 \le y \le h_1$)

$$\frac{d^2 s_1}{dy^2} - M^2 s_1 = -1 \tag{11}$$

where $M^2 = \dfrac{\rho\lambda^2 h^2}{\mu_1}$

and $\tau = \dfrac{ds_1}{dy} + B_n$ (12)

Plug flow region

Here, we take $\tau = \tau_0$ for $y_1 \le y \le y_2$

Zone II: $y_2 \le y \le h_1$

$$\frac{d^2 s_2}{dy^2} - M^2 s_2 = -1 \tag{13}$$

and $\tau = \dfrac{ds_2}{dy} - B_n$ (14)

Region II $h_1 \le y \le 1$

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$$\frac{\mu}{1+\lambda_1}\frac{d^2 s_3}{dy^2}-M^2 s_3=-1 \tag{15}$$

The non-dimensional boundary conditions are

$\tau = \tau_1$ at $y = 0$

$s_1 = 0$ at $y = 0$

$s_1(y_1) = s_2(y_2)$

$$\left.\frac{ds_1}{dy}\right]_{y=y_1}=\left.\frac{ds_2}{dy}\right]_{y=y_2}=0$$

$s_2 = s_3$ at $y = h_1$ (16)

$$\frac{ds_2}{dy}-B_n=\frac{\mu}{1+\lambda_1}\frac{ds_3}{dy} \quad \text{at } y = h_1$$

$s_3 = C_0$ at $y = 1$,

where $c_0=\dfrac{U}{u_p}$

Solution of the Problem

Solving (11) to (15), subject to the conditions (16), we obtain the velocity fields as:

$$s_1=\frac{1}{M^2}\left[M\left(\tau_1-B_n\right)\sinh My-\cosh My+1\right], \tag{17}$$

where

$$\tau_1=B_n+\frac{\operatorname{Tanh}My_1}{M}$$

Zone II:

$$s_2=c_3e^{My}+c_4e^{-My}+\frac{1}{M^2} \tag{18}$$

Zone III:

$$s_3=c_5e^{\sqrt{\frac{1+\lambda_1}{\mu}}My}+c_6e^{-\sqrt{\frac{1+\lambda_1}{\mu}}My}+\frac{1}{M^2} \tag{19}$$

where,

$$c_3=\frac{e^{-Mh_1}}{2}\left[\frac{Bn}{M}+c_5e^{\sqrt{\frac{1+\lambda_1}{\mu}}Mh_1}\left(1+\sqrt{\frac{\mu}{1+\lambda_1}}\right)+c_6e^{-\sqrt{\frac{1+\lambda_1}{\mu}}Mh_1}\left(1-\sqrt{\frac{\mu}{1+\lambda_1}}\right)\right]$$

$$c_4=\frac{e^{Mh_1}}{2}\left[c_5e^{\sqrt{\frac{1+\lambda_1}{\mu}}Mh_1}\left(1-\sqrt{\frac{\mu}{1+\lambda_1}}\right)+c_6e^{-\sqrt{\frac{1+\lambda_1}{\mu}}Mh_1}\left(1+\sqrt{\frac{\mu}{1+\lambda_1}}\right)-\frac{Bn}{M}\right]$$

$$c_5=\frac{-\frac{Bn}{M}e^{-\sqrt{\frac{1+\lambda_1}{\mu}}M}\operatorname{Cosh}M(y_2-h_1)-\left(c_0-\frac{1}{M^2}\right)e^{-\sqrt{\frac{1+\lambda_1}{\mu}}Mh_1}\left[\operatorname{Sinh}M(y_2-h_1)-\sqrt{\frac{\mu}{1+\lambda_1}}\operatorname{Cosh}M(y_2-h_1)\right]}{2\operatorname{Sinh}M(y_2-h_1)\operatorname{Sinh}\sqrt{\frac{1+\lambda_1}{\mu}}M(h_1-1)+2\sqrt{\frac{\mu}{1+\lambda_1}}\operatorname{Cosh}M(y_2-h_1)\operatorname{Cosh}\sqrt{\frac{1+\lambda_1}{\mu}}M(h_1-1)}$$

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$$c_6 = \frac{-\frac{Bn}{M}e^{\sqrt{\frac{1+\lambda_1}{\mu}}M}\operatorname{Cosh}M(y_2-h_1)-(c_0-\frac{1}{M^2})e^{\sqrt{\frac{1+\lambda_1}{\mu}}Mh_1}[\operatorname{Sinh}M(y_2-h_1)+\sqrt{\frac{\mu}{1+\lambda_1}}\operatorname{Cosh}M(y_2-h_1)]}{2\operatorname{Sinh}M(y_2-h_1)\operatorname{Sinh}\sqrt{\frac{1+\lambda_1}{\mu}}M(1-h_1)-2\sqrt{\frac{\mu}{1+\lambda_1}}\operatorname{Cosh}M(y_2-h_1)\operatorname{Cosh}\sqrt{\frac{1+\lambda_1}{\mu}}M(1-h_1)}$$

The plug velocity is given by

$$s_p = \frac{1}{M^2}\left[M(\tau_1 - B_n)\sinh My_1 - \cosh My_1\right] \tag{20}$$

and the plug range limits y_1 and y_2 can be obtained by solving the equations

$$c_3 e^{My_2} - c_4 e^{-My_2} = \frac{1}{M^2}\left[M(\tau_1 - B_n)\cosh My_1 - \sinh My_1\right] \quad \text{and}$$

$$c_3 e^{My_2} + c_4 e^{-My_2} = \frac{1}{M^2}\left[M(\tau_1 - B_n)\sinh My_1 - \cosh My_1\right] \tag{21}$$

Results and Discussions

The unsteady Couette flow of a Jeffrey fluid in contact with Bingham fluid is investigated. Here we calculate the velocities for the Jeffrey fluid and Bingham fluid in the channel. We find some interesting results observed through graphs.

From Fig. 2 we observe that as the Jeffrey parameter increases, the velocity of the Jeffrey fluid is decreasing in the upper region and the increase in the Jeffrey parameter will not effect the velocity of the Bingham fluid in the lower region. The variation of velocity with y for different Couette numbers is shown in Fig. 3. Here we observe that as the Couette number increases, the velocity of the fluid increases. From Fig. 4 we observe that as the viscosity ratio increases the velocity of the Jeffrey fluid region is increasing and the increase in the viscosity ratio will not affect the velocity of the Bingham fluid i.e. the effect of viscosity ratio on the velocity of the overall fluid comparatively less. It is noticed from Fig. 5 that as the Bingham number increases the velocity of the fluid increases.

The interesting phenomenon we discuss here is the plug velocity. From Figs. 6 and 7 we observe that the velocity increases with the increase in the Bingham number and Couette number. We notice from Fig. 8 that as the height of the interface increases the velocity of the fluid is the plug region increases. As the parameter M increases, the plug velocity is decreasing, which is shown in Fig. 9.

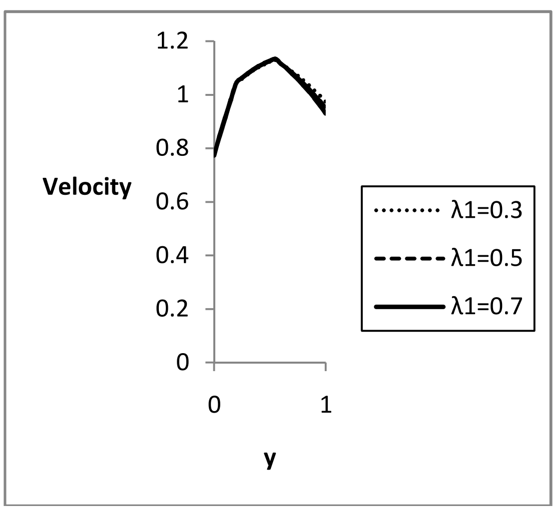

Fig 2: Variation of velocity with y for different values of the jeffrey parameter.

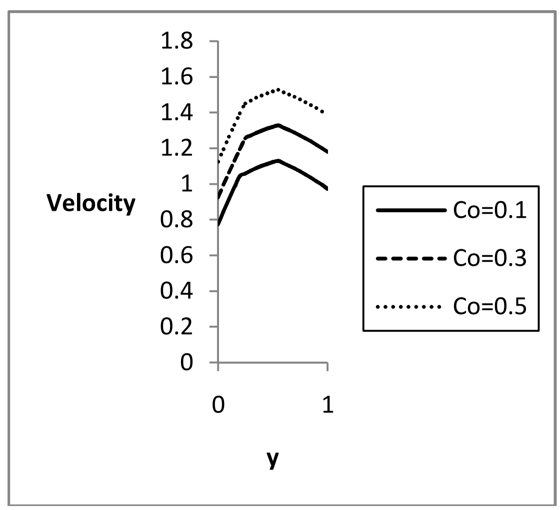

Fig 3: Variation of velocity with y for different values of the Couette Number.

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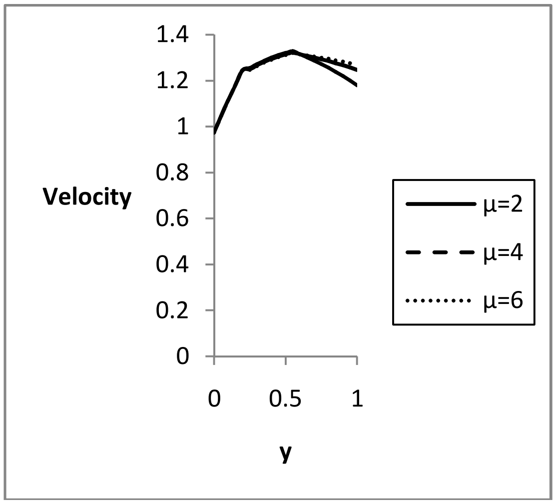

Fig 4: Variation of velocity with y for different values of the Viscosity ratio.

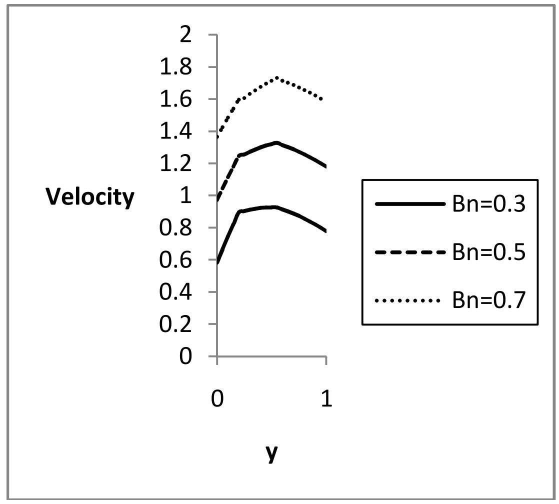

Fig 5: Variation of velocity with y for different values of the Bingham Number.

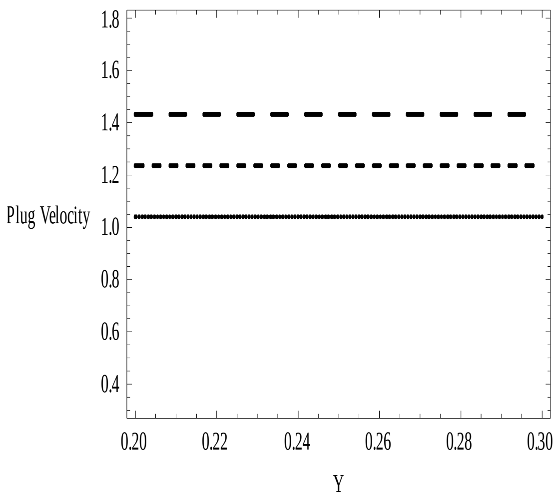

Fig 6: Variation of Plug velocity with y for different values of the Couette Number.

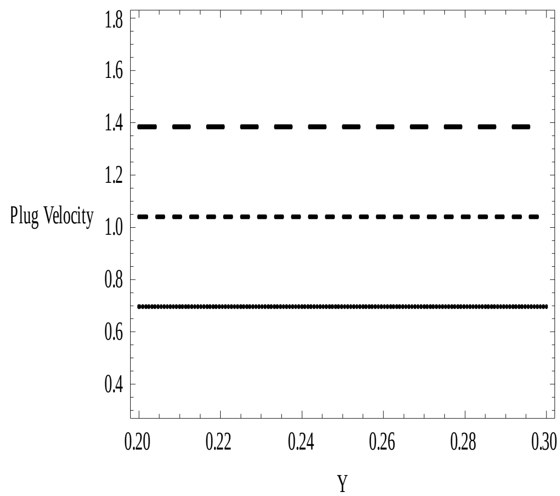

Fig 7: Variation of Plug velocity with y for different values of the bingham Number.

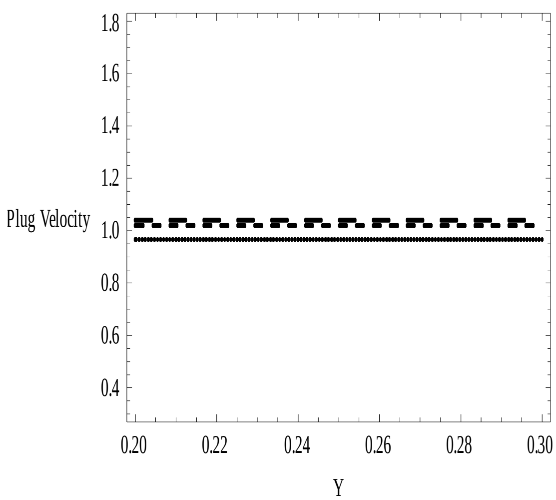

Fig 8: Variation of Plug velocity with y for different h_1 (height of the interface).

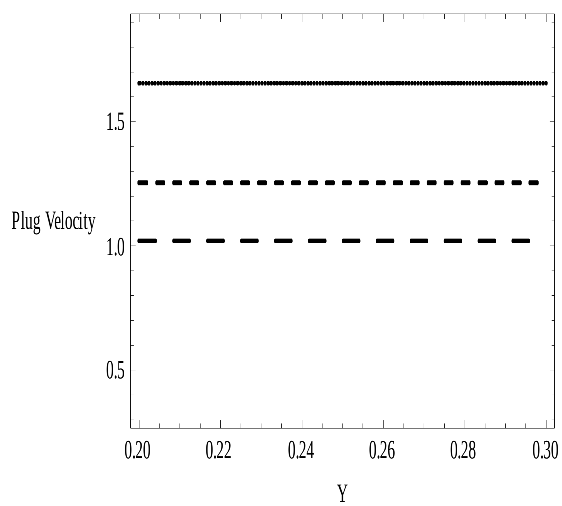

Fig 9: Variation of Plug velocity with y for different values of the M.

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